# Effect of Solar Radiation Disturbance on a Flexible Beam in Orbit

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**The uncontrolled dynamics of an orbiting flexible beam in the presence of solar radiation pressure forces is considered. The effect of solar radiation forces and moments on the rigid and flexible modes of a free-free flexible beam is evaluated. It is found that moments result only from the flexible symmetric modal deformations. For small pitch amplitudes, and at near geosynchronous altitudes, the solar radiation moments due to the deformations of the beam are seen to be greater than those due to the gravity-gradient. Within the linear range, simulated pitch steady-state amplitudes can be more significant than the induced amplitudes of the modal shape functions.**

## Nomenclature

| | |
|---|---|
| $A_n$ | = $n$th modal amplitude |
| $a_0, b_0, c_0$ | = direction cosines of the incident solar radiation vector |
| $c$ | = viscous damping coefficient |
| $c_I$ | = $J/I_d$ |
| $E_n$ | = $n$th modal force |
| $\bar{e}$ | = external force |
| $\bar{F}$ | = solar radiation pressure force |
| $h_0$ | = solar energy constant |
| $I_d$ | = dumbbell moment of inertia |
| $\bar{i}, \bar{j}, \bar{k}$ | = orthogonal unit vectors |
| $J$ | = pitch moment of inertia |
| $k$ | = torsional spring constant |
| $l$ | = length of the beam |
| $N$ | = moment due to solar radiation pressure |
| $N_m$ | = maximum moment per unit deflection |
| $M_n$ | = $n$th modal mass |
| $n$ | = mode number |
| $\bar{n}$ | = unit normal to the surface |
| $\bar{R}$ | = position vector |
| $s$ | = arc length |
| $t$ | = time |
| $x, y, z$ | = Cartesian coordinates |
| $\alpha$ | = angle between the dumbbell axis and the local vertical |
| $\delta$ | = deflection at one end of the beam |
| $\epsilon_n$ | = nondimensionalized modal amplitude = $A_n/l$ |
| $\epsilon_r$ | = coefficient of reflectivity |
| $\theta$ | = pitch angle |
| $\theta_i$ | = solar incidence angle measured relative to the normal of the undeflected beam |
| $\xi$ | = nondimensionalized $x$ coordinate = $x/l$ |
| $\tau$ | = nondimensionalized time parameter = $\omega_c t$ |
| $\bar{\tau}$ | = unit vector in the direction of incident solar radiation |
| $\phi$ | = beam shape function |
| $\Omega_n$ | = $n$th modal frequency |
| $\omega_c$ | = orbital angular velocity |
| $\omega_n$ | = nondimensionalized $n$th modal frequency = $\Omega_n/\omega_c$ |

*Subscripts*

| | |
|---|---|
| $a$ | = absorbing surface |
| $r$ | = reflecting surface |
| $\epsilon r$ | = surface with coefficient of reflectivity, $\epsilon_r$ |

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## I. Introduction

PROPOSED future applications of large space structures require control of the shape and orientation of the structure. These structures will be designed so that their overall moments of inertia will correspond to those required for gravitational stabilization. The major environmental disturbance acting on these structures at their proposed operational altitudes will then be due to the solar radiation pressure and thermal effects. The solar radiation pressure effects can become very important for certain geometric shapes and material properties of the flexible structure. Therefore, it is necessary to evaluate the solar radiation pressure effects on these large, inherently flexible space structures in orbit before control of their shape and orientation can be implemented.

The effects of solar radiation pressure on rigid spacecraft structural systems have been studied previously by a number of authors. A partial summary of these investigations is included in Refs. 1-5. Notable among these are Refs. 3 and 4 which propose the utilization of the rotations of rigid control surfaces (relative to the satellite) interacting with the solar radiation pressure in order to provide attitude control torques.

In this paper it is proposed to study the disturbance effects due to solar radiation pressure on a basic flexible structural element in orbit, namely, a long flexible free-free beam. The equations of motion for a free-free flexible beam nominally oriented along the local vertical were developed previously.[6] Later, the work of Ref. 6 was extended to consider the motion and stability of the beam about a nominal local horizontal orientation. This system included a rigid dumbbell used for gravitational stabilization that was assumed to be connected to the center of mass of the beam through a gimballed passive damping device.[7] The control aspects of such a beam using point actuators were also considered in Ref. 8. The effect of solar radiation pressure on the dynamics of these two types of beam structures is studied here, and to the authors' knowledge, represents the first time that solar disturbance torques acting on large flexible space systems have been treated.

The force and moment expressions obtained by Karymov[1] are used to develop the solar radiation disturbance model for a beam by considering the individual mode shapes of the free-free beam. The transverse elastic displacements are assumed to be small so that the shadowing of the beam due to any deflected part of the beam can be neglected.

## II. Determination of Solar Radiation Forces and Moments Acting on a Flexible Beam

Let the direction of the incident solar radiation $\bar{\tau}$ in the body coordinate system be denoted as

$$\bar{\tau}=a_0\bar{i}+b_0\bar{j}+c_0\bar{k} \tag{1}$$

and let $\bar{n}$ be the outward unit vector normal to the surface d$s$ of a body of arbitrary shape exposed to solar radiation (Fig. 1). Then, the solar radiation force acting on a completely absorbing surface $\bar{F}_a$ and that acting on a completely reflecting surface $\bar{F}_r$ can be obtained as[1]

$$\bar{F}_a=-h_0\bar{\tau}\int_s \bar{\tau}\cdot\bar{n}\,\mathrm{d}s \tag{2}$$

$$\bar{F}_r=-2h_0\int_s \bar{n}(\bar{\tau}\cdot\bar{n})^2\mathrm{d}s \tag{3}$$

where, $h_0=4.6\times 10^{-6}$ N/m$^2$ is a constant for Earth-orbiting spacecraft. It is clear from Eqs. (2) and (3) that the solar radiation force acts in the direction of the incident solar radiation vector for a completely absorbing surface and for a completely reflecting surface the solar radiation force is in a direction opposite to the normal to the surface at the point of incidence of solar radiation. The integration over an area $s$ is bounded by the condition

$$\bar{\tau}\cdot\bar{n}\leq 0 \tag{4}$$

The corresponding moments for a completely absorbing surface $\bar{N}_a$ and for a completely reflecting surface $\bar{N}_r$ can be developed as[1]

$$\bar{N}_a=h_0\bar{\tau}\times\int_s \bar{R}(\bar{\tau}\cdot\bar{n})\,\mathrm{d}s \tag{5}$$

$$\bar{N}_r=2h_0\int_s \bar{n}\times\bar{R}(\bar{\tau}\cdot\bar{n})^2\mathrm{d}s \tag{6}$$

where $\bar{R}$ is the position vector of d$s$ with respect to the center of mass. For a surface with an arbitrary reflection coefficient $\epsilon_r$, the force and moment expressions become[1]

$$\bar{F}_{\epsilon r}=\bar{F}_a+\epsilon_r(\bar{F}_r-\bar{F}_a) \tag{7}$$

$$\bar{N}_{\epsilon r}=\bar{N}_a+\epsilon_r(\bar{N}_r-\bar{N}_a) \tag{8}$$

The forces and moments due to solar radiation pressure acting on a free-free flexible beam can now be obtained by considering the shape function of the beam $\phi$ (Fig. 1, only the first antisymmetric mode is depicted). The beam is assumed to vibrate in the transverse direction only so that the normal at any point is given by

$$\bar{n}=(\phi'\bar{i}-\bar{k})/\sqrt{1+\phi'^2} \tag{9}$$

where $\phi'=\mathrm{d}\phi/\mathrm{d}\xi$ and $\xi$ is the nondimensionalized longitudinal coordinate of the beam with the elemental length,

$$\mathrm{d}s=\mathrm{d}\xi\sqrt{1+\phi'^2} \tag{10}$$

If the analysis is restricted to a single plane containing $\xi$ and $\zeta$, $\bar{\tau}$ reduces to

$$\bar{\tau}=a_0\bar{i}+c_0\bar{k} \tag{11}$$

Using Eqs. (9-11) in Eq. (2), the total force acting per unit width of the beam is expressed as

$$\begin{aligned}\bar{F}_a&=-h_0\bar{\tau}\int_0^l (a_0\bar{i}+c_0\bar{k})\cdot(\phi'\bar{i}-\bar{k})\,\mathrm{d}\xi\\ &=a_0c_0h_0\bar{i}+h_0c_0^2\bar{k}\ \text{(for symmetric modes)}\\ &=-h_0(2a_0\delta_0-c_0)(a_0\bar{i}+c_0\bar{k})\ \text{(for asymmetric modes)}\end{aligned} \tag{12}$$

where, $\delta_0=\phi_z^n(0)=$ deflection at one end of the beam for the $n$th mode. The total force per unit width of the beam acting on a completely reflecting surface is obtained after substituting Eqs. (9-11) into Eq. (3) as

$$\bar{F}_r=-2h_0\int_0^l \frac{(a_0\phi'-c_0)^2}{(1+\phi'^2)}(\phi'\bar{i}-\bar{k})\,\mathrm{d}\xi \tag{13}$$

The expressions for the moments per unit width of the beam are developed using Eqs. (9-11) in Eqs. (5) and (6) as

$$\begin{aligned}\bar{N}_a&=-h_0\bar{\tau}x\left[\int_0^l (a_0\phi'-c_0)\left\{\left(\xi-\frac{l}{2}\right)\bar{i}+\phi\bar{k}\right\}\right]\mathrm{d}\xi\\ &=-h_0a_0c_0\delta_0\ \text{(for symmetric modes)}\\ &=0\ \text{(for asymmetric modes)}\end{aligned} \tag{14}$$

$$\begin{aligned}\bar{N}_r&=2h_0\int_0^l \frac{(a_0\phi'-c_0)^2}{(1+\phi'^2)}(\phi'\bar{i}-\bar{k})\times\left\{\left(\xi-\frac{l}{2}\right)\bar{i}+\phi\bar{k}\right\}\mathrm{d}\xi\\ &=-2h_0\int_0^l \frac{(a_0\phi'-c_0)^2}{(1+\phi'^2)}\left\{\phi'\phi+\left(\xi-\frac{l}{2}\right)\right\}\bar{j}\,\mathrm{d}\xi\end{aligned} \tag{15}$$

Equations (13) and (15) involve complicated line integrals. These integrals can be evaluated using numerical integration methods. For the purpose of this numerical study a beam of length 100 m with tip deflections of 0.01$l$ and 0.1$l$ were considered. Figure 2 shows the variation of the resultant horizontal and normal force components of a beam with a completely absorbing surface as the solar incidence angle $\theta_i$ is varied from 0 to 90 deg. Here, $\theta_i$ represents the angle between the normal to the undeflected beam and $\bar{\tau}$. The horizontal and normal force components are measured relative to the beam's undeflected axes. As expected, for small tip deflections of the beam, the resultant horizontal absorbing force component becomes zero for incidence angles of 0 and 90 deg, respectively, while the normal component has a maximum am-

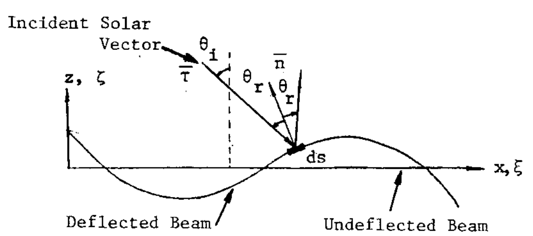


**Fig. 1 Geometry of reflection of a flexible beam exposed to solar radiation.**

plitude at zero incidence angle. In Fig. 2 and subsequent figures the individual effect of each mode, with the assumed beam tip deflection as indicated in the figure, is illustrated. Figures 3a and 3b show force distributions along the beam length for the first two beam flexible modes and for both completely absorbing and completely reflecting surfaces (with the angle of incidence taken to be 45 deg). The asymmetric nature of the force distribution gives rise to a resultant moment about the mass center for a symmetric beam mode. An asymmetric mode of the beam is seen to have a symmetric force distribution and, therefore, has a zero net moment about the beam's mass center.

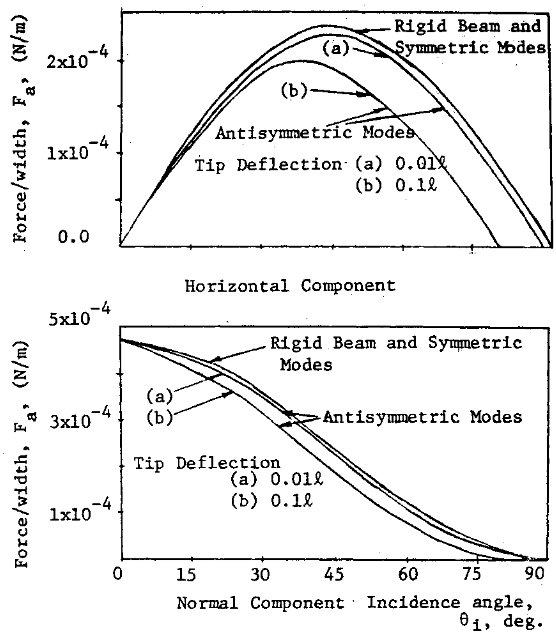


**Fig. 2 Variation of solar force components with incidence angle: totally absorbing surface, free-free beam (length $l = 100$ m).**

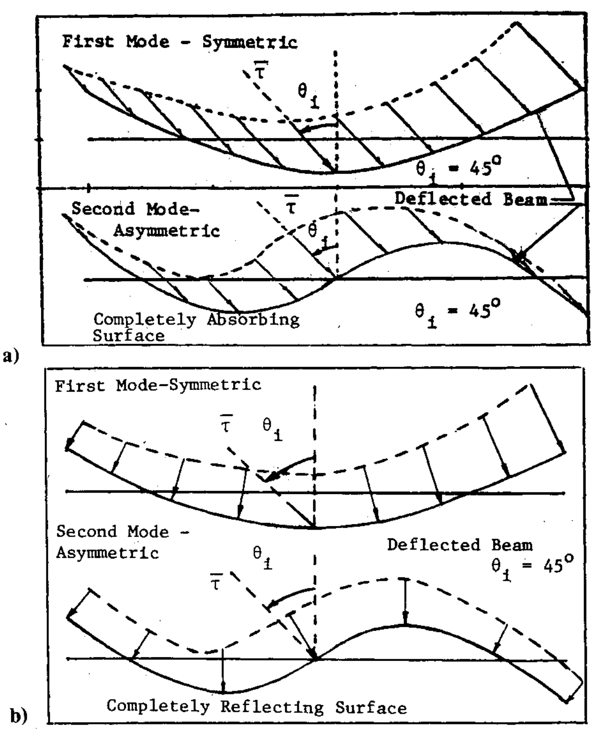


**Fig. 3 Solar radiation force distribution on the first two modes, free-free beam.**

The magnitude of the resultant moments as the solar incidence angle is varied is shown in Fig. 4 for each symmetric mode and the assumed tip deflection. Large moments can result for larger deflections whereas these moments would be zero for a rigid beam. Because the force distribution for an asymmetric mode is symmetric about an axis passing through the mass center and parallel to the incident solar radiation, the moments for all asymmetric modes are zero (Fig. 3). For small pitch angle displacements, the moment due to solar radiation pressure may become greater than the moment due to the gravity-gradient forces as shown in Fig. 5. It is seen that at geosynchronous altitudes, the moment due to solar radiation may become predominant even for deflections of the order of $0.01l$.

Figures 6 and 7 show the forces and moments for a completely reflecting surface, obtained using numerical integrating techniques based on Eqs. (3) and (6). It is seen that the moment for the completely reflecting case increases with the larger value of the tip deflection. Since the radiation force acts along the normal to the surface for a completely reflecting surface, and the deflections of the beam are assumed small, the normal force components are seen to be much greater than the horizontal force components (Fig. 6). Further, the resultant horizontal force components also depend on the mode shapes (Fig. 6) in contrast to the case of the completely absorbing surface (Fig. 2). Other results (not shown) for larger tip deflections ($\delta = 0.1$) indicate this dependency on the particular mode shape to be more pronounced for both the horizontal and the normal force components. The moments for the reflecting beam also depend on the specific mode number of the beam incorporated into the model as shown in Fig. 7. Because of the symmetric force distribution about the center of mass the resultant moment is zero for all asymmetric modes, as before. Hence, the moments are zero for all asymmetric modes regardless of the surface reflectivity. In the case of a reflecting beam the force distribution is independent of the slopes along the length of the beam for the case of small deflections, but for higher modes and larger deflections the slopes along the length of the beam become larger. Hence, both the force distribution and the resultant moment are seen to be

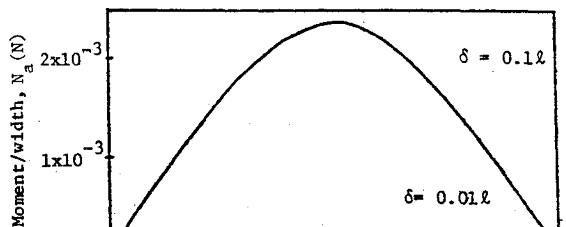


**Fig. 4 Pitch moment due to solar radiation pressure, completely absorbing surface; effect of each symmetric mode in the system.**

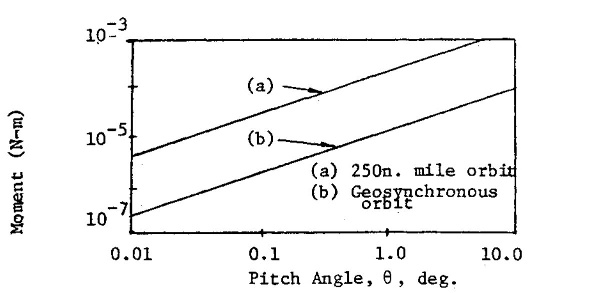


**Fig. 5 Moment due to gravity-gradient force as a function of pitch angle (100 m rigid beam).**

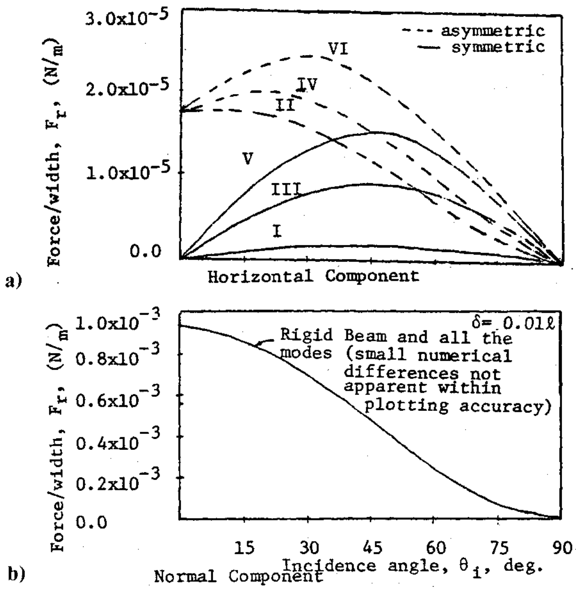


Fig. 6 Variation of solar force components with incidence angle, totally reflecting surface.

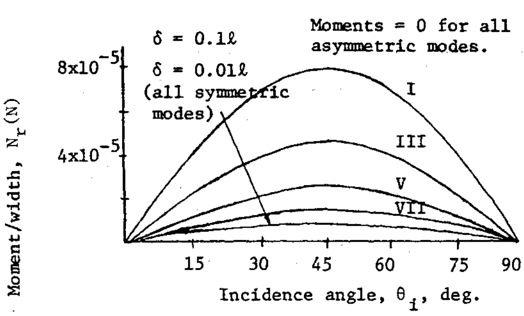


Fig. 7 Pitch moment due to solar radiation pressure, completely reflecting surface.

dependent on the specific mode number for large deflections of the reflecting beam.

With the aid of these moment diagrams, it is now possible to model the disturbance torque due to solar radiation pressure, once the number of modes and the associated modal deflections are specified. This aspect is considered in the next section.

## III. Solar Radiation Disturbance Model

A beam nominally oriented along the local horizontal or local vertical is considered. Such a beam makes one revolution per orbit with respect to the incident solar radiation. For any symmetric mode and for a given coefficient of reflectivity $\epsilon_r$, the pitch torque can be expressed as a function of the solar incidence angle $\theta_i$ in the form (from Figs. 4 and 7),

$$N \propto N_m \sin\theta_i \cos\theta_i \tag{16}$$

where

$$N_m = N_{a_m} + \epsilon_r (N_{r_m} - N_{a_m})$$

and $N_{a_m}, N_{r_m}$ are the maximum moment per unit deflection for a completely absorbing surface (from Fig. 4) and for a completely reflecting surface (from Fig. 7), respectively.

For small deflections, $N$ is proportional to the deflection at one end of the beam, $\delta(t)$, and $\delta(t)$ is also a function of $A_n(t)$, the $n$th modal amplitude function [where $A_n(t) = \epsilon_n(t) l$].

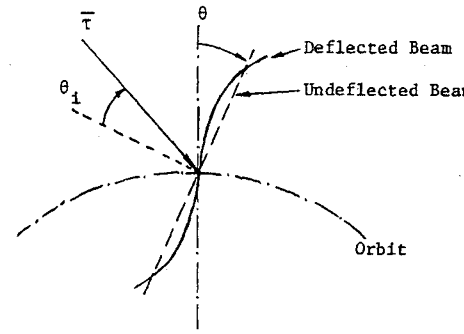


Fig. 8 A flexible beam nominally oriented along the local vertical.

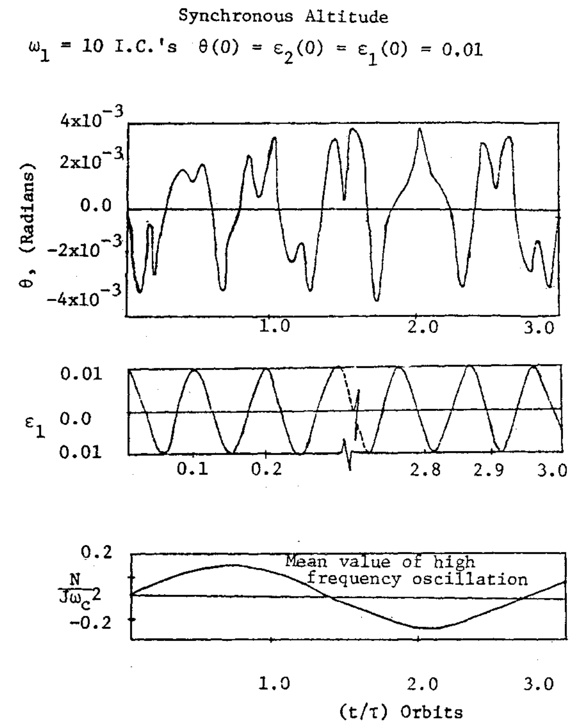


Fig. 9 Time response of the beam nominally along the local vertical and in the presence of solar radiation pressure.

Thus,

$$N(t) = \epsilon_n(t) N_m l \sin\theta_i \cos\theta_i \tag{17}$$

and $\theta_i$ is given by

$$\theta_i(t) = \omega_c t + \theta(t) + \theta_i(0) \tag{18}$$

where $\omega_c$ is the orbital angular velocity, and $\theta$ the pitch angle of the beam. The effect of the disturbance on the generic mode is obtained by evaluating the integral[6]

$$E_n = \int \bar{\phi}^{(n)}(\xi) \cdot \bar{F} \mathrm{d}s \tag{19}$$

where $\bar{F}\mathrm{d}s$ is the differential force due to solar pressure. Equations (2) and (3) are substituted into Eq. (19) to obtain

$$E_{n_a} = \int \bar{k}\phi_z^{(n)} \cdot \{h_0 \bar{\tau}(\bar{\tau}\cdot\bar{n})\} \mathrm{d}s$$

$$= h_0 c_0^2 \int_0^l \phi_z^{(n)} \mathrm{d}\xi = 0 \tag{20}$$

and

$$E_{n_r} = \int \bar{k}\phi_z^{(n)} \cdot \{-2h_0 \bar{n}(\bar{\tau}\cdot\bar{n})^2\} \mathrm{d}s$$

$$= 2h_0 c_0^2 \int_0^l \phi_z^{(n)} \mathrm{d}\xi = 0 \quad (21)$$

After combination of Eqs. (19) and (20), the generic force is obtained as

$$E_n = E_{n_a} + \epsilon_r (E_{n_r} - E_{n_a}) = 0 \quad (22)$$

For $\epsilon_r = 0.5$ and a tip deflection of $0.01l$, Eq. (16) yields

$$N_m = 2.23\times 10^{-4} + 0.5(0.4\times 10^{-5} - 2.23\times 10^{-4})$$

$$= 1.58\times 10^{-4}\ \text{N-m}$$

This is the maximum torque that is experienced by the beam for a unit deflection equal to 1 m in a 100 m length beam at any instant in the orbit. The generic forces are seen to be zero for the assumed small deflections of the beam.

## IV. Effect of Solar Radiation Pressure on a Flexible Beam Nominally Oriented along the Local Vertical

The equations of motion for a thin uniform beam in orbit with its axis nominally along the local vertical (Fig. 8) is developed in Ref. 6. The beam is assumed to undergo only inplane angular motions and deformations and it is assumed, also, that the center of mass of the beam follows a circular orbit. The beam's elastic motions are considered to be unconstrained and the longitudinal vibrations of the beam are assumed to be negligible in comparison with the transverse vibrations. For the case of small amplitude pitch oscillations of the beam, the linearized equations of motion are derived as[6]

$$\theta'' + 3\theta = N/J\omega_c^2 \qquad \epsilon_n'' + \omega_n^2\epsilon_n = E_n/M_n l\omega_c^2 \quad (23)$$

Only the first two flexural modes of the beam will be included in the analysis. Using Eqs. (17) and (22) and the numerical value for $N_m$, the following three equations of second order result, based on $\omega_c$ for a geosynchronous orbit.

$$\theta'' + 3\theta = 3.6\epsilon_l \sin\theta_i \cos\theta_i, \quad \epsilon_l'' + \omega_l^2\epsilon_l = 0, \quad \epsilon_2'' + \omega_2^2\epsilon_2 = 0 \quad (24)$$

The first- and second-modal oscillations are seen to be decoupled from each other and the pitch motion. Therefore, $\epsilon_l$ and $\epsilon_2$ have solutions of the form

$$\epsilon_l = c_l \sin\omega_l\tau + c_2\cos\omega_l\tau, \quad \epsilon_2 = c_3\sin\omega_2\tau + c_4\cos\omega_2\tau \quad (25)$$

where $c_l$, $c_2$, $c_3$, and $c_4$ are constants to be determined from the initial conditions. The pitch equation now becomes

$$\theta'' + 3\theta = 1.8\ (c_l\sin\omega_l\tau + c_2\cos\omega_l\tau)\sin 2\theta_i$$

Assuming $\theta_i(0) = 0$ and $\theta(t)$ very small, $\theta_i(t) = \omega_c t = \tau$ from Eq. (18). With $\epsilon_l(0) = \epsilon_0$ and $\epsilon_l' = 0$, $c_l = 0$ and $c_2 = \epsilon_0$, and the pitch equation becomes

$$\theta'' + 3\theta = 0.9\epsilon_0\{\sin(2+\omega_l)\tau + \sin(2-\omega_l)\tau\} \quad (26)$$

The solution of this equation can be obtained in the form

$$\theta(\tau) = c_5\sin\sqrt{3}\tau + c_6\cos\sqrt{3}\tau + \frac{0.9\epsilon_0}{3-p^2}\sin p\tau - \frac{0.9\epsilon_0}{3-q^2}\sin q\tau$$

where $p = 2+\omega_l$ and $q = 2-\omega_l$.

With $\theta(0) = 0$, $\epsilon_l(0) = 0.01$, and $\omega_l = 10$, the pitch response is given by

$$\theta(\tau) = 0.002392\sin\sqrt{3}\tau + 0.000638\sin 12\tau - 0.001475\sin 8\tau \quad (27)$$

The response of the beam to the solar radiation disturbance obtained using numerical integration of Eq. (24) is shown in Fig. 9. The pitch motion shown in Fig. 9 is identical with the response obtained using Eq. (27) and shows a maximum pitch amplitude of 0.23 deg. The effect of the disturbance on the first-modal oscillations is seen to be negligible.

## V. Effect of Solar Radiation Pressure on a Dumbbell Stabilized Flexible Beam Nominally Oriented along the Local Horizontal

The uncontrolled local horizontal orientation of the beam represents an unstable motion. This unstable configuration of the beam can be stabilized by using a rigid dumbbell such that the resulting gravity-gradient torques provide stabilization. In Ref. 7, the equations of motion for a beam with a dumbbell assumed to be attached at the center of mass of the beam (Fig. 10) through a spring-loaded hinge and having viscous rotational damping have been developed. In addition to the assumptions made in developing Eqs. (23), it is further assumed that the dumbbell mass is concentrated at the tips and that the viscous force at the hinge is linear. With the usual assumptions of small pitch amplitude and dumbbell oscillations and flexural deformations, the linearized equations of motion in the absence of active control and external forces are obtained as[7]

$$\theta'' + \bar{c}\theta' + (\bar{k}-3)\theta - \bar{c}\alpha' - \bar{k}\alpha + \sum_n (\bar{c}\epsilon_n' + \bar{k}\epsilon_n) C_z^{(n)} = \frac{N}{J\omega_c^2} \quad (28)$$

$$\alpha'' + c_I\bar{c}\alpha' + (c_I\bar{k}+3)\alpha - c_I\bar{c}\theta' - c_I\bar{k}\theta - \sum_n (\bar{c}\epsilon_n' + \bar{k}\epsilon_n) c_I C_z^{(n)} = 0 \quad (29)$$

$$\epsilon_n'' + (\omega_n^2 - 3)\epsilon_n - \{\bar{k}(\alpha-\theta) + \bar{c}(\alpha'-\theta')\} C_z^{(n)} \left(\frac{J}{M_n l^2}\right) + \sum_m (\bar{c}\epsilon_m' + \bar{k}\epsilon_m) C_z^{(mn)} = \frac{E_n}{M_n\omega_c^2 l} \quad (30)$$

where $C_z^{(mn)} = JC_z^{(m)}C_z^{(n)}/M_n l^2$; $(m,n = 1,2,\ldots)$; $M_n$ is the mass of the beam for all $n$; and

$\bar{k} = k/J\omega_c^2$; $\bar{c} = c/J\omega_c$

$C_z^{(n)} = \left.\dfrac{\partial\phi_z^{(n)}}{\partial x}\right|_{x=0}$

$\phi_z^{(n)}$ = beam shape function of the $n$th transverse mode

$c_I = J/I_d$; $I_d$ = pitch moment of inertia of the dumbbell

As before, only the first two modes will be considered. The forcing terms are the same as for the case of the beam along the local vertical, Eq. (24). Since the dumbbell is assumed to be rigid, there is no net moment acting on the dumbbell due to solar radiation pressure. The first mode influences the pitch motion through the forcing function and the second mode affects the dumbbell motion through coupling. Thus, pitch, dumbbell, and the two modes of the beam are all coupled to each other and the resulting system of equations are too complicated to yield analytical solutions. These equations were numerically integrated with initial tip deflections of $0.01\ell$ in the first mode and zero initial displacements in $\theta$, $\alpha$, and $\epsilon_2$, respectively (Fig. 11). The steady-state response shows

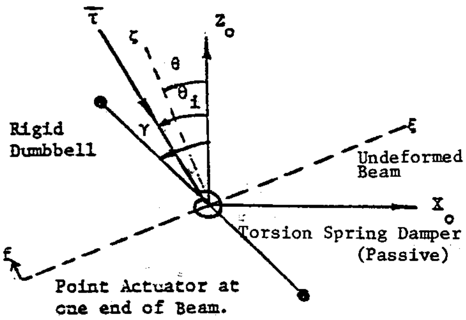


**Fig. 10 Dumbbell stabilized flexible beam nominally oriented along the local horizontal with passive and active controllers.**

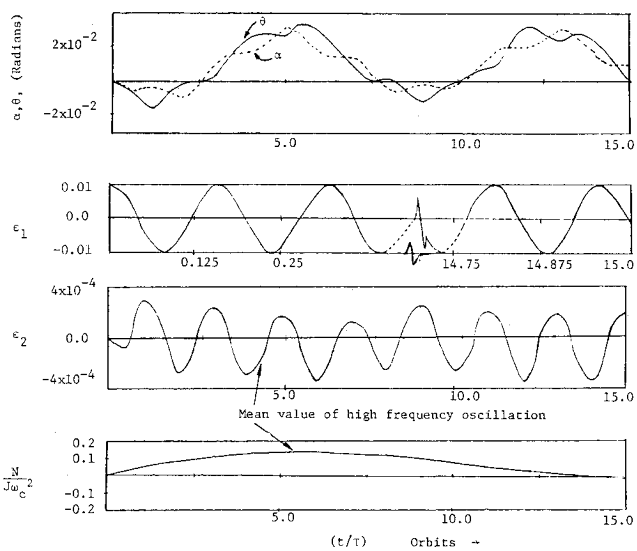


**Fig. 11 Time response of dumbbell stabilized flexible beam in the presence of solar radiation pressure ($\omega_f = 10.0$).**

pitch amplitudes as high as 2 deg. The first-modal oscillations are not greatly affected due to the solar radiation pressure. The second mode is excited because of the dumbbell motion, but the amplitude remains small (maximum $|\epsilon_2| = 0.002$). The high-frequency oscillations in $\epsilon_2$ and in the pitch acceleration, $N/J\omega_c^2$, are suppressed in Fig. 11 for the sake of simplicity.

A similar response for a stiffer beam with $\omega_f = 20.0$ and the same initial conditions and beam parameters as for the case in Fig. 11 was obtained. The maximum pitch amplitude was seen to be about 0.23 deg, one order of magnitude less than that for the beam with $\omega_f = 10.0$. In this case the higher frequencies in the second mode damp the pitch oscillation, through the dumbbell motion, more rapidly so that the pitch amplitudes do not build up. Once again, $\epsilon_1$ and $\epsilon_2$ motions are not affected to first order because of the solar radiation pressure. Thus, the effect of solar radiation pressure is seen to affect mainly the pitch motion.

Figure 12 shows the effect of solar radiation pressure on a beam which is at a low altitude Earth orbit (460 km). The pitch excitation is seen to be very small (0.005 deg), as expected, because at the low altitudes the gravity-gradient torques are predominant (Fig. 5).

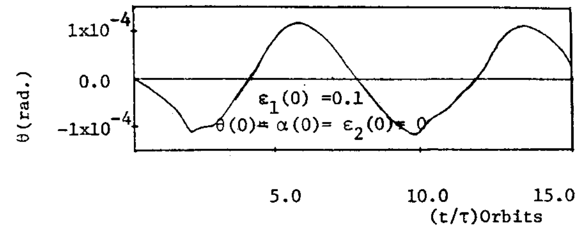


**Fig. 12 Pitch response of dumbbell stabilized flexible beam in the presence of solar radiation pressure at a low-altitude orbit (460-km orbit).**

## VI. Conclusions

A solar radiation disturbance model for a large flexible beam in orbit is obtained. At geosynchronous altitudes the moments due to solar radiation pressure on a flexible beam can be larger than the moments due to the gravity-gradient. Simulated steady-state dynamic responses, for the beam parameters and deflections considered here, indicate that the induced rigid (pitch) amplitudes can be more significant than the induced flexible modal amplitudes. An evaluation of the effect of the solar radiation disturbance due to flexibility of the beam on the previously obtained active control laws (developed without considering environmental torques) is suggested as a follow-on effort. A study of the dynamics of large flexible plates and more complex three-dimensional structures under the influence of solar radiation pressure is also proposed. The effects of the thermal gradients due to the solar radiation on the dynamics of basic flexible structural elements may also be significant.

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